2020

APPENDICES

Building economic resilience in Queensland

Appendix A: Lockdown modelling

This appendix describes the technical construction of the susceptible–infected–recovered ('SIR') model used for some simulations. The model does not attempt to forecast epidemiological or economic events. Rather, it was used to help understand the motivations of policymakers, and having understood the motivation of policy makers, broadly describe policy options going forward. The modelling was originally undertaken as a preliminary part of this research, before the announcement of the staged reduction in lockdowns. This staged lockdown reduction was in line with modelling outcomes, strengthening the model's credibility.

The model was adapted from (Alvarez et al. 2020).

A.1 General description

In the model, the policymaker has two objectives

- to minimise the loss of life, measured in present value statistical value of life (SVL) terms, and
- to minimise the loss of output, including that due to the death of a member of the population.

The policymaker can pursue the first outcome by shutting down some economic activity, which reduces the spread of the virus. A reduction in the transmission rate has two effects

- it prevents some people from catching the disease at all, and
- it reduces the mortality rate among those who are infected, as it prevents the healthcare system from being overloaded.

Regardless of the policymaker's choice, the population will adapt to the 'new normal' over time (for example, private individuals and businesses choosing to wear masks). Similarly, economic agents will adapt to economic restrictions, increasing their effectiveness. These improvements do not happen indefinitely, however; the underlying transmission rate and 'efficiency of the lockdown' are bounded below and above, respectively.

The modelling tends to result in two broad types of policy:

- A strong initial response to massively suppress the infection rate, which slowly tapers off as infections dwindle and the population adapts. Some level of restriction continues indefinitely to prevent an outbreak until a vaccine can be found.[1] The policymaker here is attempting to minimise new infections to minimise the loss of life.
- A weaker initial response accompanying the first wave of infections, which ends completely after that wave has passed. In this model, the policymaker intervenes to prevent an increased death rate associated with an overloaded hospital system but allows the virus to spread throughout the population until it is halted by herd immunity.

In general, most jurisdictions have attempted to pursue the first strategy initially, with mixed success. Those that fail in achieving the first, appear to 'fall back' onto the second.

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[1] While a vaccine is not actually 'discovered' in the model, the probability of developing and deploying a vaccine is used to weight the present value expected cost of the strategy.

A.2 Technical details

Population dynamics

The model is run over discrete days (indexed by t). For each day the initial population is split into being susceptible (S_t), infected (I_t), recovered (R_t) or dead (D_t).[2] The model is normalised such that total population is 1.

All but a minute proportion of the population start as susceptible. The number of susceptible people that become infected is given by

$$\dot{S} = -\underbrace{\beta_t}_{\text{underlying rate}} * \underbrace{[S_t(1-\theta_t L_t)]}_{S_t \text{ in contact}} * \underbrace{[I_t(1-\theta_t L_t)]}_{I_t \text{ in contact}}$$

β_t represents the underlying rate of transmission of the disease. Unlike in the original paper, our model allows β to be time variant, causing the underlying infection rate to fall (to a lower bound) as people adopt anti-infection measures on their own.

L_t is a value between 0 and $\min\{1/\theta_t, \bar{L}\}$ representing the proportion of the economy locked down. θ_t represents the effectiveness of the lockdown in reducing the transmission of the disease. Unlike the original paper, θ is time variant, to allow the economy to adapt to the shutdown, increasing the effectiveness of the lockdown over time (to an upper bound).

The change in the infected population is given by

$$\dot{I} = \underbrace{-\dot{S}}_{\text{newly infected}} - \underbrace{\gamma I_t}_{\text{recovered or died}}$$

where $1/\gamma$ is the expected time a person remains infected before recovering or dying.

The change in R and D are given by, respectively

$$\dot{D} = \gamma I_t * \underbrace{\phi(I_t)}_{\text{moratlity rate}} \text{ and } \dot{R} = \gamma I_t * \underbrace{\left(1-\phi(I_t)\right)}_{\text{survival rate}}$$

$\phi(I_t)$ represents the death rate for those who are infected, as a function of the number of infected. In our model, the death rate rises with infection to represent the increased health system burden until it reaches its upper bound (the mortality rate without treatment).

The social planner's problem

The daily flow cost (c_t), to the social planner, on day t is given by and has components:

$$c_t = \underbrace{L_t w * [S_t + I_t + (1-\tau)R_t]}_{\text{cost of the lockdown}} + \underbrace{I_t \phi(I_t)\chi}_{\text{cost of deaths}}$$

The 'cost of the lockdown' is simply the proportion of the economy shutdown multiplied by GDP, w, (normalised to per capita). τ is a variable between 0 and 1 which determines the proportion of those recovered who are locked down; if $\tau = 1$, then recovered people are not affected by the lockdown, and continue to be economically productive. The 'cost of death' term is simply the deaths on the day multiplied by the cost per death, χ.

The optimal lockdown policy is given by minimising the present value expected social cost function;

$$C = \sum_{t=0}^{T} c_t * e^{-(r+v)t}$$

[2] This is in contrast to the description in the paper, where 'recovered' included both alive and dead.

Where r is the daily interest rate, and v is the daily chance of developing a vaccine.

The 'optimal' lockdown policy is then given by

$$L^* = \underset{\mathrm{L}}{\operatorname{argmin}}\, C(L) : \theta_t L_t \in [0,1]\ \forall\, t \in T$$

Unlike the original paper, we impose the additional constraint that the lockdown policy is set in 30-day blocks.

The optimisation process

Sequential quadratic programming is used to solve for the socially optimal policy given the set of parameters and initial conditions required.

Because the lockdown policy settings are monthly while the time dynamics are daily, the objective function, with a time horizon of $T = 30N$ days, operates as follows:

- An input vector $L_m^p = (L_0, \dots, L_N)$ of monthly lockdown policies is transformed to a daily set of lockdown policies, $L_d^p = (L_0, \dots L_0, \dots., L_N, \dots, L_N)$, where each $L_{n \in N}$ occurs 30 times.
- The cost of that policy is calculated as $C(L_d^p)$.

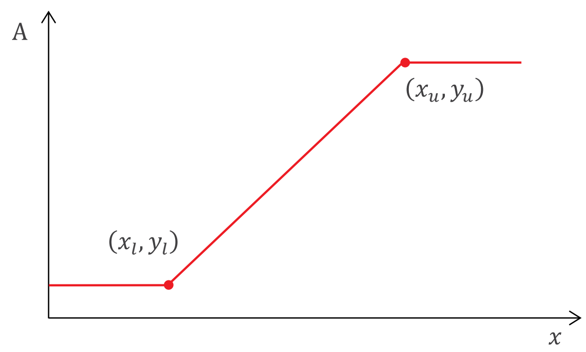
Piecewise parameters

A piecewise (A) function is used to define θ and β in terms of time, and ϕ in terms of infection. This function is defined as:

$$\mathrm{A}: \mathbb{R}^5 \to \mathbb{R}; (x, x_l, x_u, y_l, y_u) \to \begin{cases} y_l \text{ if } x < x_l \\ (y_h - y_l)\dfrac{x - x_l}{x_u - x_l} + y_l \text{ if } x \in [x_l, x_u] \\ y_u \text{ if } x > x_u \end{cases}$$

This function was used where there were lower and upper threshold values, above and below which the function was roughly constant. For example, the mortality rate is roughly constant below a certain level of infection, rises as the health system is overwhelmed, before stabilising at the higher rate (the rate without treatment).

Figure A.2 Plot of A with respect to x

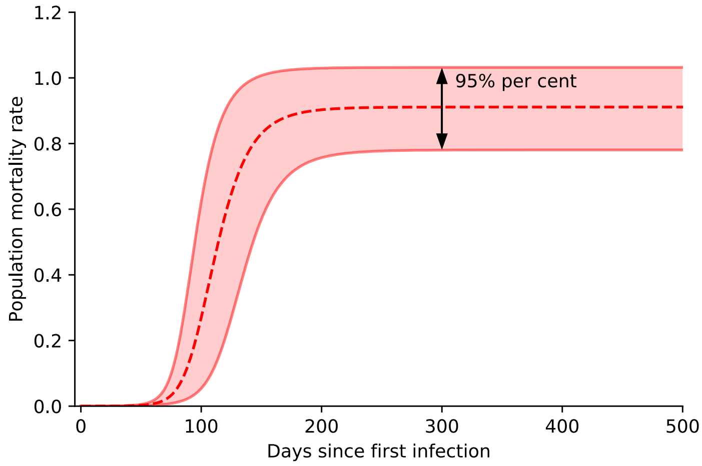


Sensitivity

The model is highly sensitive to certain parameter assumptions. Small changes in the infection and mortality rates significantly influence the spread of and deadliness of the virus.

A simple demonstration of this is given in Figure A.3, which shows 95 percent confidence range of total population mortality over time (in the absence of a policy response) if the underlying infection transmission rate β was normally distributed with a mean of 0.15 and a variance of 0.01.

Figure A.3 Distribution of mortality rates over time, without policy intervention



While the model is relatively less sensitive to changes in the statistical value of life and the effectiveness of the shutdown, these values are associated with a much greater degree of uncertainty.

Appendix B: Small area employment impacts — technical note

Small area employment impacts were calculated by combining the employment impacts by industry and by SA4[3] (ABS 2020, cat. no. 6291.0.55.003) with the counts of person employed in each industry group (by SA3 place of work) collected in the 2016 census. This process distributes the job losses in an SA4 across the SA3s that comprise it based on the industry composition of those SA3 and the state-wide employment impacts on each industry.

The value ($\hat{v}$) given to each SA3 (a) was calculated as (Census data are in blue, labour force data are in red)

$$\hat{v}^{\,a} = \frac{\sum_{i \in I} w_i \, n_i^a}{\sum_{i \in I} n_i^a} \times \frac{\sum_{i \in I} \sum_{a \in A} n_i^a}{\sum_{i \in I} \left(w_i \sum_{a \in A} n_i^a \right)} \times v^A$$

Where

- i is an industry and I is the set of industries,
- w_i is the Queensland wide change in employment for industry i,
- n_i^a is the 2016 census count of the number of persons employed in SA3 a in industry i, and
- v^A is the published change in jobs in SA4 A.

[3] SA3s and SA4s are different levels of 'statistical areas' defined in the ABS's statistical geography. An SA4 is formed by aggregating SA3s. More information can be found in ABS 2020, cat. no. 1270.0.55.003.

Appendix C: Experimental estimates of the speed of adjustment

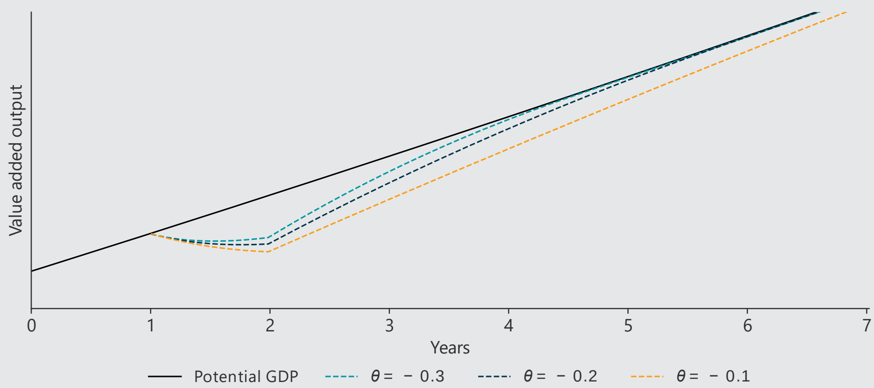
The purpose of the modelling was to understand the rate at which the Queensland and National economies recover after an economic shock. Two models were considered, the first using labour productivity as the dependent variable, and the second using value-added output as the dependent variable, described below. A panel dataset was constructed consisting of 16 market sector industries for each state and territory, with annual time series for each variable from 1989–90 or 1994–95 depending on the industry.

Box C.1 gives a brief overview of error correction models.

Box C.1 The error correction model (ECM)

An ECM models how long it takes a variable (in this output) to return to its long-run trend after a deviation. The parameter of interest is the proportional reduction of the error per time period. In other words an error correction term of $-\theta$ indicates that in each period, 100θ per cent of the current difference between actual and potential GDP is recovered.

Figure C.1 Example of different error correction terms on recovery



C.1 Overview

An error correction model (ECM) is used to estimate the rate at which cointegrated variables return to their long-run equilibrium after a deviation or shock. The model can be written as

$$\Delta y_{it} = \delta\Delta \mathrm{x}_{it} + \theta_i\left[y_{i(t-1)} - \beta_i \mathrm{x}_{it}\right] + \epsilon_{it}$$

where

- y_{it} is the dependent variable for industry i in year t
- $\Delta y_{it} = y_{it} - y_{i(t-1)}$ is the change in y from the previous year to year t,
- x_{it} is a vector of explanatory variables,
- $\Delta \mathrm{x}_{it}$ is the change in those variables from the previous year to year t,
- δ and β are estimated parameters,
- ϵ_{it} is an error term, and
- θ_i is the error correction term of interest.

All x and y are expressed in natural log terms, such that Δx is a percentage change.

Two models were considered. In the first, y is value-added output, and x consists of capital, labour, the capital-to-labour ratio and MFP. In the second, y is labour productivity and x consists of capital-to-labour and MFP.

C.2 Data

The data was all drawn from ABS data sources (2019, cat. nos. 5220.0, 5206.0, 6291.0.55.003).

It covers each state[4] and is cross-sectional across the 16 market sector industries. *Rental, hiring and real estate services, professional scientific and technical services, administrative and support services* and *other services* are from 1994–95 to 2018–19. All other industries are from 1989–90 to 2018–19.

Labour inputs were constructed on an hours worked basis. Capital inputs were based on QPC constructed capital service measures for each state and each market sector industry. Net Capital Stocks (NKS) shares for each state are used to allocate national Productive Capital Stocks (PKS). NKS are confidentialised by the ABS for certain industries for some states (QPC 2020). Where this is the case, the industry for that state is dropped creating an unbalanced panel.

Multi-factor productivity (MFP) indices were constructed for each industry for each state using standard growth accounting methodologies.[5] Basic summary statistics are found in Table C.2.

[4] A reference to 'state' in this section includes the ACT and NT.
[5] Further information can be found in QPC (2020) as well as ABS 2007 cat. no. 5260.0.55.001,

Table C.2 State dataset, summary statistics

Variables (in logs)	Obs	Mean	Std. Dev.	Min	Max
Output (value added)	3,840	4.240	0.436	2.397	5.447
Capital services	3,400	4.099	0.576	0.682	5.788
Labour input (hours worked)	3,840	4.476	0.347	2.601	6.606
MFP	3,038	4.515	0.351	3.009	6.172
Labour productivity	3,840	4.125	0.724	2.171	7.707
Capital-Labour ratio	3,267	4.286	0.505	0.596	6.063

Cross-sectional dependence

Not accounting for unobserved dependence between cross-sectional units causes the error term to be autocorrelated and leads to biased ordinary least-squares (OLS) regression results.

Tests indicate that strong cross-sectional dependence exists for each variable. Weak cross-sectional dependence is rejected. The results of these tests are presented in Table C.3.

Table C.3 Cross-sectional dependence tests

Test	Labour productivity	Capital-labour ratio	MFP	Output	Capital	Labour (hrs worked)
All states						
Pesaran (2004) CD test[1] H_0: Panels are cross-sectional independent	137.7 (0.000)	173.0 (0.000)	52.7 (0.000)	265.8 (0.000)	290.0 (0.000)	91.7 (0.000)
Pesaran (2015) CD test[2] H_0: Errors are weakly cross-sectional dependent	492.0 (0.000)	174.3 (0.000)	120.7 (0.000)	492.4 (0.000)	278.2 (0.000)	492.5 (0.000)
All states						
Pesaran (2004) CD test[1] H_0: Panels are cross-sectional independent	32.3 (0.000)	45.2 (0.000)	11.7 (0.000)	55.0 (0.000)	54.4 (0.000)	26.0 (0.000)
Pesaran (2015) CD test[2] H_0: Errors are weakly cross-sectional dependent	59.9 (0.000)	43.2 (0.000)	18.6 (0.000)	59.9 (0.000)	51.5 (0.000)	59.9 (0.000)

Note: [1] Pesaran (2004) CD tests for cross-sectional independence. [2] Chudik & Pesaran (2015) tests for weak cross sectional dependence.

The residual multifactor approach to dealing with cross-sectional dependence in panels where N is large assumes that the cross dependence can be characterized by a small number of unobserved common factors, possibly due to economy-wide shocks that affect all units, albeit with different intensities.

Stationarity and co-integration

A stationary process has a time-invariant mean and a time-invariant variance. A nonstationary process has a time-varying mean, a time-varying variance, or both. When the first difference of a nonstationary process is stationary, the process is said to be integrated of order one, denoted $I(1)$. When a linear combination of several $I(1)$ series is stationary, the series are said to be cointegrated (Engle & Granger 1987). Testing for cointegration is a process for testing whether in fact the variables have a long-run equilibrium relationship. The variables may wander arbitrarily for a period of time, but the variables return to their long-run relationship if they are indeed cointegrated.

Unit root tests confirm the order of integration of the data—whether the variables are stationary $I(0)$ or of a higher order $I(\geq 1)$. Ipshin, Fisher and Hadri Lagrange Multiplier tests indicate that the state panel data are non-stationary. The same test results are found when tests are conducted on Queensland market sector industries only. These results are summarised in Table C.4.

Table C.4 Univariate unit root tests

Test	Labour productivity	Capital-labour ratio	MFP	Output	Capital	Labour (hrs worked)
All states						
Ipshin[1], (lags = 2) H_0: all panels contain unit roots H_1: some panels are stationary	–1.403 (0.756)		– 1.514 (0.275)	– 1.414 (0.714)		– 1.425 (0.665)
Fisher (ADF)[2], (lags = 2) H_0: all panels contain unit roots H_1: at least one panel is stationary	231.9 (0.858)	208.0 (0.710)	223.0 (0.257)	241.7 (0.721)	236.3 (0.483)	252.0 (0.559)
Hadri Lagrange Multiplier test[3] H_0: all panels are stationary H_1: some panels contain unit roots	34.4 (0.000)		42.2 (0.000)	69.4 (0.000)		30.0 (0.000)
Queensland						
Ipshin[1], (lags = 1) H_0: all panels contain unit roots H_1: some panels are stationary	–1.742 (0.164)		– 1.626 (0.320)	– 1.497 (0.538)		– 1.576 (0.401)
Fisher (ADF)[2], (lags =1) H_0: all panels contain unit roots H_1: at least one panel is stationary	22.021 (0.907)	19.206 (0.964)	31.838 (0.475)	34.751 (0.338)	33.176 (0.410)	30.318 (0.552)
Hadri Lagrange Multiplier test[3] H_0: all panels are stationary H_1: some panels contain unit roots	15.596 (0.000)		21.768 (0.000)	37.568 (0.000)		21.930 (0.000)

Note: [1] T-bar statistic with p-values in brackets. [2] Augmented Dickey Fuller (ADF) form of test. Test statistic with p-value in brackets. For each variable, the null is also not rejected for forms of the test where the null assumes a trend or random walk with non-zero drift. [3] Z(tau) statistic with p-value in brackets for test assuming heteroskedastic disturbances across units. The null is also rejected in all cases with the alternative assumptions of homoskedastic disturbances and serial dependence in the errors.

There are very strong theoretical reasons for expecting co-integration in production functions and labour productivity models. Empirical tests on the dataset used in this paper support cointegration, although the evidence is stronger for the Australian panel than the Queensland panel (Table C.5). This is likely due to the greater power of the tests when applied to the larger Australian panel (much greater number of observations). Combined with greater test power, it may also be related to the relatively larger role of mining in Queensland. There are significant capital services and MFP measurement issues in this industry that could affect the test statistics as the alternative hypothesis requires all panels to be cointegrated in order to reject the null of no cointegration. Less stringent panel cointegration tests all supported cointegration. The estimated models below produce ECM terms that are negatively signed, consistent with economic theory, and statistically significant providing further evidence of cointegration.

Table C.5 Co-integration tests

Test	Production function	Labour productivity
All states		
Kao		
Modified Dickey-Fuller t	2.745 (0.003)	–0.285 (0.388)
Augmented Dickey-Fuller t	3.657 (0.000)	1.168 (0.121)
Pedroni		
Modified Phillips-Perron t	3.744 (0.000)	–1.974 (0.024)
Augmented Dickey-Fuller t	-1.099 (0.135)	–5.343 (0.000)
Westerlund		
Variance ratio	3.204 (0.001)	–1.441 (0.075)
Queensland		
Kao		
Modified Dickey-Fuller t	0.203 (0.420)	–0.516 (0.303)
Augmented Dickey-Fuller t	0.400 (0.345)	0.270 (0.394)
Pedroni		
Modified Phillips-Perron t	1.752 (0.040)	1.544 (0.061)
Augmented Dickey-Fuller t	0.046 (0.482)	0.207 (0.418)
Westerlund		
Variance ratio	2.448 (0.007)	2.307 (0.011)

Note: For all tests, H_0 is that there is no co-integration, H_1 is that all panels are co-integrated. Series are demeaned to mitigate the impacts of cross-sectional dependence. P-values are in brackets.

C.3 Estimation

Estimation was performed in Stata with the *xtdcce2* command using Pooled Mean Groups (PMG). Results for the two models, and their significance, are given in Table C.6.

Table C.6 Production and labour productivity function estimates

	Model A1		Model A2		Model Q1		Model Q2	
Dataset	All states		All states		Queensland		Queensland	
Dep. Variable	ΔOutput (VA)		ΔLP		ΔOutput (VA)		ΔLP	
Short-run –								
$\Delta Capital_t$	0.316 (0.020)	***			0.279 (0.071)	***		
$\Delta Labour_t$	0.528 (0.019)	***			0.490 (0.052)	***		
$\Delta K/L_t$			0.329 (0.020)	***			0.291 (0.054)	***
ΔMFP_t	0.830 (0.006)	***	0.851 (0.005)	***	0.709 (0.018)	***	0.764 (0.014)	***
ECM term (θ)	–0.150 (0.052)	***	–0.132 (0.046)	***	–0.249 (0.103)	**	–0.184 (0.079)	***
Long-run –								
$Capital_t$	0.359 (0.042)	***			0.284 (0.100	***		
$Labour_t$	0.397 (0.053)	***			0.479 (0.130)	***		
K/L_t			0.541 (0.037)	***			0.509 (0.124)	***
MFP_t	0.918 (0.032)	***	0.982 (0.034)	***	0.958 (0.063)	***	1.067 (0.048)	***
Constant								
Tests -								
Units	121		121		16		16	
Observations	3196		3074		410		393	
F - statistic	77.0		265.4		30.0		131.6	
Pesaran CD test^	10.1 (0.000)		12.46 (0.000)		0.92 (0.356)		1.40 (0.163)	

*Note: MFP estimates for each state and territory are based on a capital services measure (see QPC 2020 for a discussion of how the capital services measures were constructed). *** indicates statistical significance at the 1 per cent level. ** indicates significance at 5 per cent. * indicates significance at 10 per cent. ^ Test for weak cross-sectional dependence (see Chudik & Pesaran 2015). Tests indicate the presence of strong cross-sectional dependence in the Australian models and weak cross-sectional dependence in the Queensland models.*
Source: QPC estimates.

References

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